

## CHAPTER 14 - FREQUENCY RESPONSE

List of topics for this chapter :

Transfer Function  
Bode Plots  
Series Resonance  
Parallel Resonance  
Passive Filters  
Active Filters  
Scaling

### TRANSFER FUNCTION

**Problem 14.1** Given the circuit in Figure 14.1 and  $i(t) = I \cos(\omega t)$  amps, find the transfer function  $H(\omega) = V_o / I$  and sketch the frequency response.

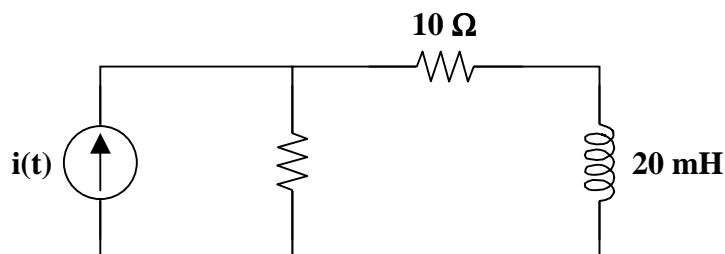
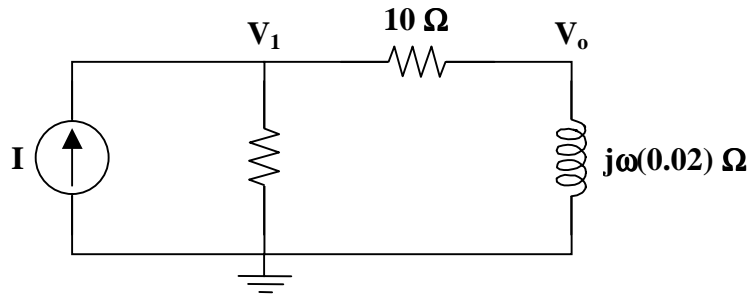


Figure 14.1

- **Carefully DEFINE the problem.**  
Each component is labeled completely. The problem is clear.
- **PRESENT everything you know about the problem.**  
To obtain the transfer function,  $H(\omega)$ , we need to obtain the frequency-domain equivalent of the circuit by replacing resistors, inductors, and capacitors with their impedances  $R$ ,  $j\omega L$ , and  $1/j\omega C$  respectively. Then, use any circuit technique to obtain  $H(\omega)$ . The frequency response of the circuit can be obtained by plotting the magnitude and phase of the transfer function as the frequency varies.
- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**  
After transforming the circuit from the time domain to the frequency domain, we can use nodal analysis, mesh analysis, or basic circuit analysis to find the transfer function. Let's examine the frequency-domain equivalent circuit in order to make the best choice.

Transforming the circuit to the frequency domain yields



It seems obvious that nodal analysis will yield two equations with three unknowns ( $I$ ,  $V_1$ , and  $V_o$ ). These equations can be manipulated to eliminate  $V_1$  in order to find  $H(\omega)$ .

➤ **ATTEMPT a problem solution.**

Using nodal analysis,

$$\text{At node 1 :} \quad -I + \frac{V_1 - 0}{10} + \frac{V_1 - V_o}{10} = 0$$

$$\text{At node 2 :} \quad \frac{V_o - V_1}{10} + \frac{V_o - 0}{j\omega(0.02)} = 0$$

Using the equation for node 1, find  $V_1$  in terms of  $V_o$  and  $I$ .

$$10I = 2V_1 - V_o$$

$$V_1 = \frac{V_o + 10I}{2} = \frac{V_o}{2} + 5I$$

Simplify the equation for node 2.

$$j\omega(V_o - V_1) + 500V_o = 0$$

$$(j\omega + 500)V_o - j\omega V_1 = 0$$

Now, substitute the equation for  $V_1$  into the simplified equation for node 2.

$$(j\omega + 500)V_o - j\omega(0.5V_o + 5I) = 0$$

$$(j\omega/2 + 500)V_o = j\omega 5I$$

$$\frac{V_o}{I} = \frac{j\omega 5}{j\omega/2 + 500} = \frac{j\omega 10}{1000 + j\omega}$$

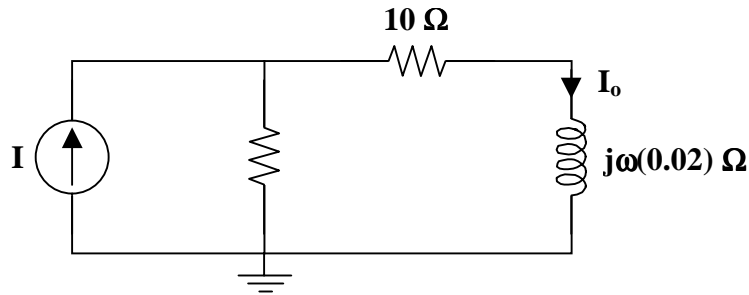
Therefore,

$$H(\omega) = \frac{j\omega 10}{1000 + j\omega}$$

➤ **EVALUATE the solution and check for accuracy.**

A check of our solution can be done using basic circuit analysis. Find the output voltage as the current through the inductor multiplied by the impedance of the inductor, i.e.,  $V = IZ$ .

Label the necessary variables for this technique.



$$I_o = \frac{10}{10 + 10 + j\omega(0.02)} I = \frac{10}{20 + j\omega(0.02)} I$$

$$V_o = ZI = \frac{(j\omega)(0.02)(10)}{20 + j\omega(0.02)} I = \frac{j\omega 10}{1000 + j\omega} I$$

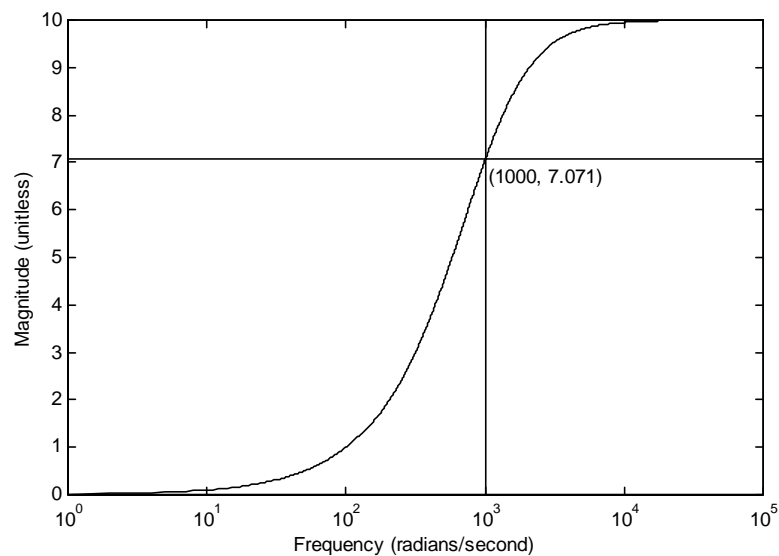
$$\frac{V_o}{I} = \frac{j\omega 10}{1000 + j\omega}$$

Our check for accuracy was successful.

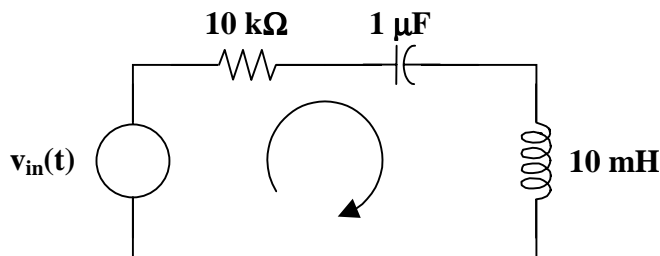
- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to “ALTERNATIVE solutions” and continue through the process again.**  
This problem has been solved satisfactorily.

$$H(\omega) = \frac{j\omega 10}{1000 + j\omega}$$

**The frequency response is**

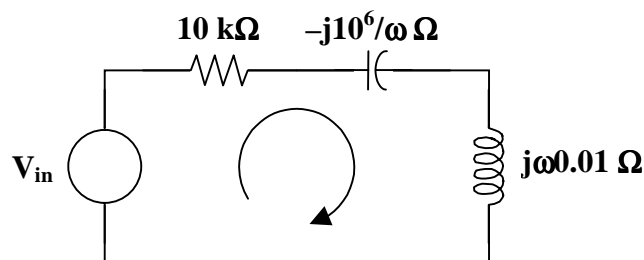


**Problem 14.2** Given the circuit in Figure 14.1 and  $v_{in}(t) = V_{in} \cos(\omega t)$  volts, find the transfer function  $H(\omega) = V_{out} / V_{in}$  and sketch the frequency response.



**Figure 14.1**

Transform the circuit to the frequency domain.



Clearly,

$$V_{out} = j\omega(0.01) I$$

and

$$I = \frac{V_{in}}{10^4 - \frac{j10^6}{\omega} + j\omega(0.01)}$$

Thus,

$$V_{in} = \left( 10^4 - \frac{j10^6}{\omega} + j\omega(0.01) \right) I$$

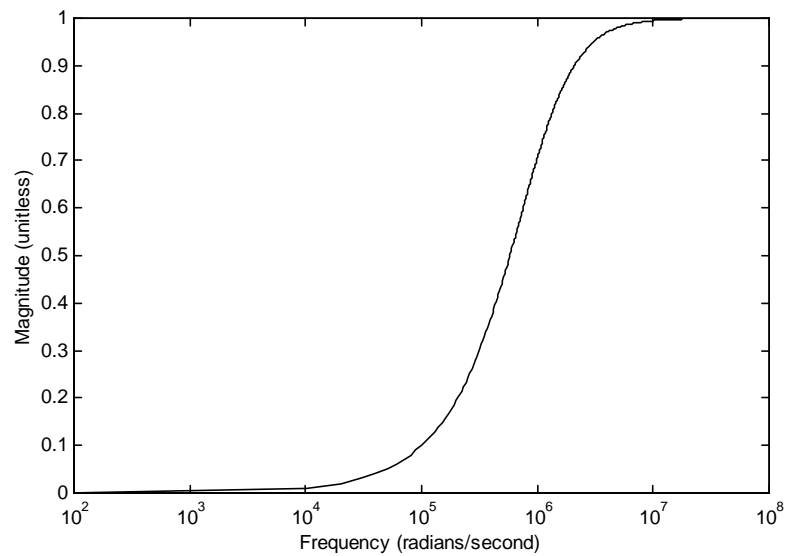
So,

$$\frac{V_{out}}{V_{in}} = \frac{j\omega(0.01) I}{\left( 10^4 - \frac{j10^6}{\omega} + j\omega(0.01) \right) I} = \frac{j\omega}{10^6 + j\left( \omega - \frac{10^8}{\omega} \right)}$$

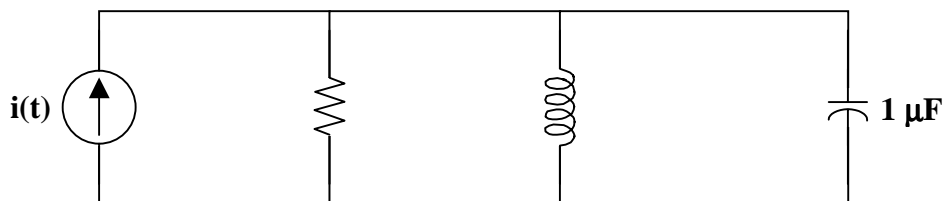
Therefore,

$$H(\omega) = \frac{j\omega}{10^6 + j\left( \omega - \frac{10^8}{\omega} \right)}$$

**The frequency response is**

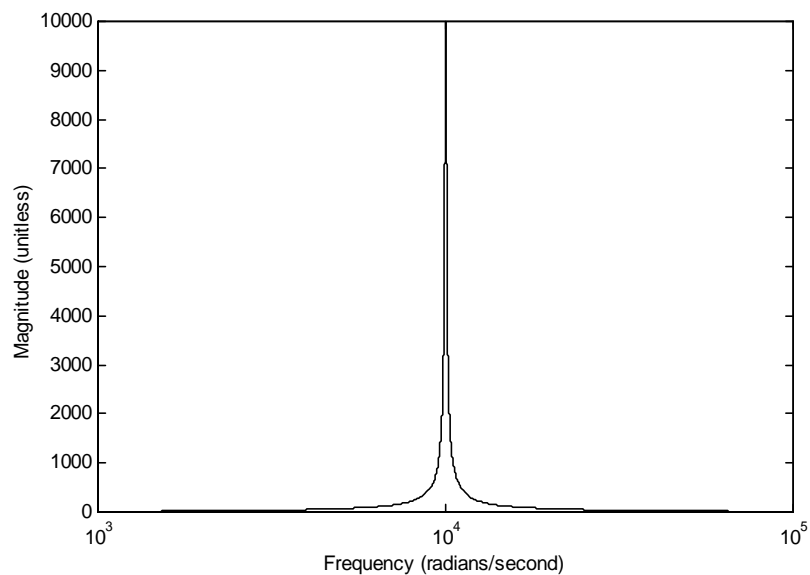


**Problem 14.3** Given the circuit in Figure 14.1 and  $i(t) = I_{in} \cos(\omega t)$  amps, find the transfer function  $H(\omega) = V_{out} / I_{in}$  and sketch the frequency response.



**Figure 14.1**

**The frequency response is**



for the transfer function

$$H(\omega) = \frac{10^4}{1 + j\left(\frac{\omega}{10^2} - \frac{10^6}{\omega}\right)}$$

## BODE PLOTS

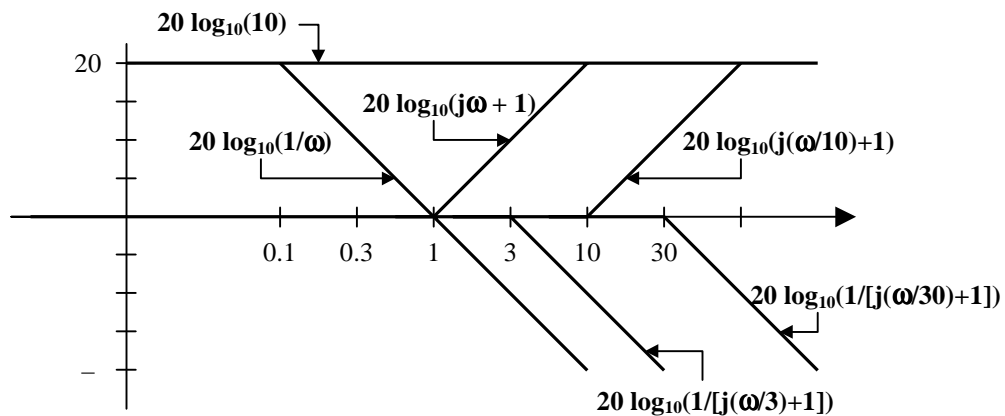
**Problem 14.4** Sketch the Bode plots, both magnitude and phase, given the following transfer function in the s-domain.

$$H(s) = \frac{(90)(s+1)(s+10)}{s(s+3)(s+30)}$$

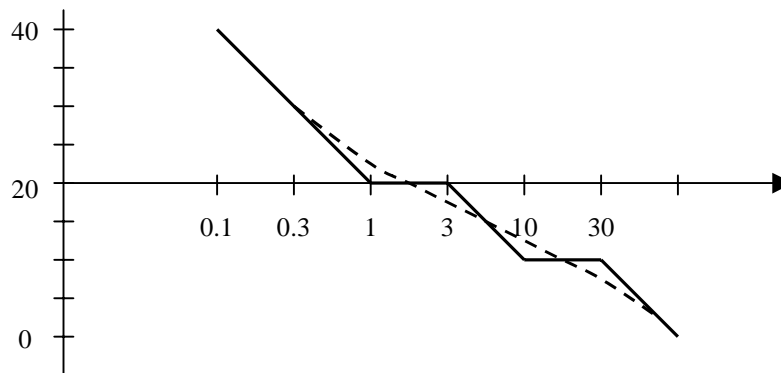
First, we need to modify the transfer function so that it is in a form that is easy to plot.

$$H(s) = \frac{(90)(s+1)(s+10)}{s(s+3)(s+30)} = \frac{(90)(10)\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)}{(3)(30)(s)\left(\frac{s}{3}+1\right)\left(\frac{s}{30}+1\right)} = \frac{(10)\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)}{(s)\left(\frac{s}{3}+1\right)\left(\frac{s}{30}+1\right)}$$

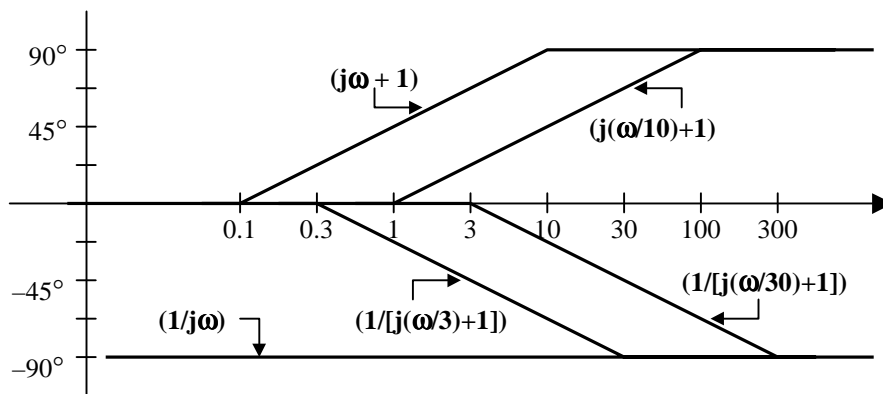
Begin with a plot showing the magnitude curve of each term in the transfer function.



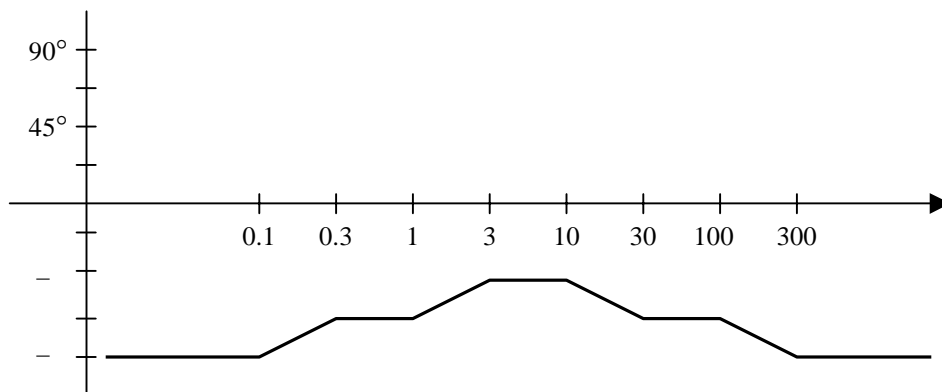
Now, combine, or add, the curves to acquire the composite **magnitude (dB) plot** of the transfer function. Note that the dashed curve shows the approximation to the actual curve.



The individual phase angle plots for each component of the transfer function are shown below.

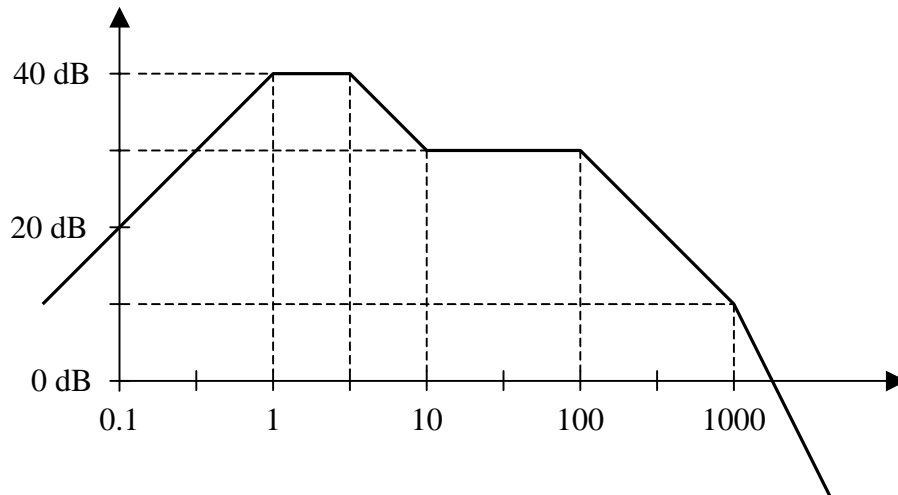


Finally, the composite **phase angle plot** for the transfer function can be drawn.



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**Problem 14.5** Given the following Bode plot, determine the value of the transfer function,  $H(s)$ , represented by the Bode plot.



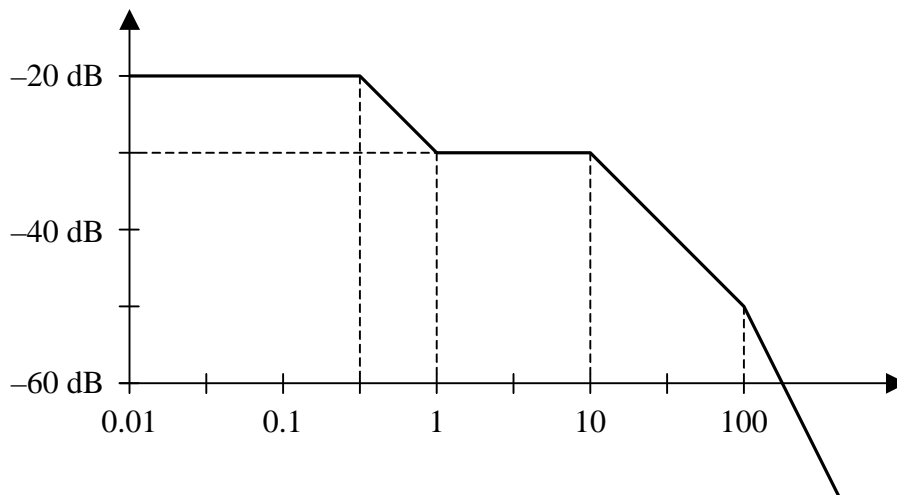
From this figure we have the following  $H(s)$ , determined by looking at each break point and realizing that the slope everywhere is incremented by 20 dB per decade. Please note that the one break point between  $\omega = 1$  and 10 is estimated to be equal to 3.

$$H(s) = \pm K \frac{s \left( \frac{s}{10} + 1 \right)}{(s+1) \left( \frac{s}{3} + 1 \right) \left( \frac{s}{100} + 1 \right) \left( \frac{s}{1000} + 1 \right)}$$

where  $K$  is given by  $20\log_{10}(K) = 40 \longrightarrow K = \underline{\underline{100}}$

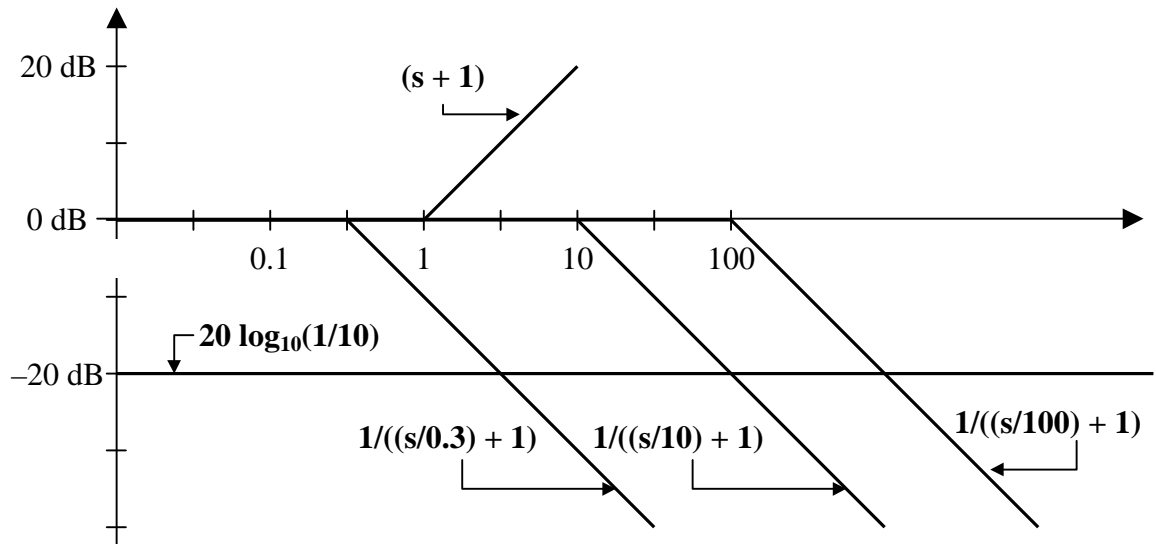
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**Problem 14.6** Given the following Bode plot, determine the value of the transfer function,  $H(s)$ , represented by the Bode plot.





Decomposing the Bode plot into its basic elements we get,



Therefore,

$$H(s) = \pm \frac{s + 1}{\left( \frac{s}{0.3} + 1 \right) \left( \frac{s}{10} + 1 \right) \left( \frac{s}{100} + 1 \right)}$$

## SERIES RESONANCE

**Problem 14.7** Given the circuit in Figure 14.1, find:

- the resonant frequency and the half-power frequencies,
- the bandwidth and the quality factor,
- the amplitudes of  $v_C(t)$  at  $\omega_0$ ,  $\omega_1$ , and  $\omega_2$ .

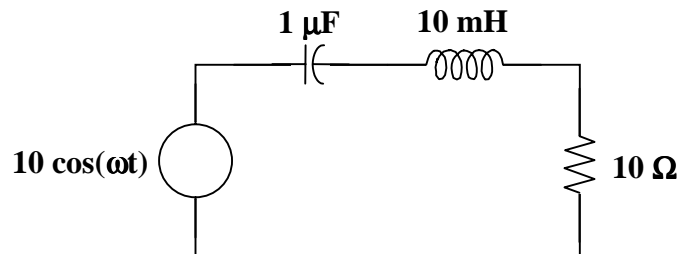


Figure 14.1

- (a) The resonant frequency of a series RLC circuit is  $\omega_0 = \frac{1}{\sqrt{LC}}$ .

So,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-6}}} = 10^4 \text{ rad/s}$$

$$\omega_0 = \underline{\underline{\mathbf{10 \text{ krad/s}}}}$$

The half-power frequency of a series RLC circuit is  $\omega_{1,2} = \mp \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$ .

So,

$$\omega_{1,2} = \mp \frac{10}{2 \times 10^{-2}} + \sqrt{\left(\frac{10}{2 \times 10^{-2}}\right)^2 + \left(\frac{1}{10^{-2} \times 10^{-6}}\right)}$$

$$\omega_{1,2} = \mp 500 + \sqrt{(500)^2 + 10^8}$$

$$\omega_{1,2} = \mp 500 + 10,012 \text{ rad/s}$$

or

$$\omega_1 = \underline{\underline{\mathbf{9.512 \text{ krad/s}}}}$$

$$\omega_2 = \underline{\underline{\mathbf{10.512 \text{ krad/s}}}}$$

- (b) The bandwidth of a series RLC circuit is  $B = \omega_2 - \omega_1$ .

So,

$$B = 10.512 - 9.512$$

$$B = \underline{\underline{\mathbf{1 \text{ krad/s}}}}$$

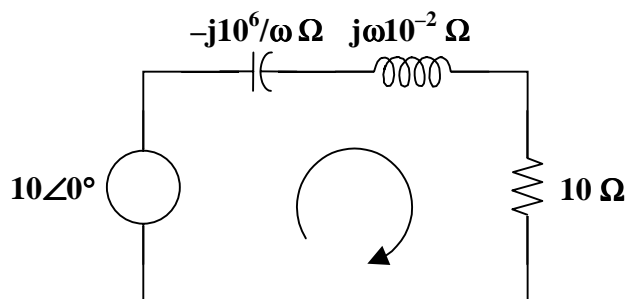
The quality factor of a series RLC circuit is  $Q = \frac{\omega_0}{B}$ .

So,

$$Q = \frac{10^4}{10^3}$$

$$Q = \underline{\underline{\mathbf{10}}}$$

- (c) The amplitudes of  $v_C(t)$  at  $\omega_0$ ,  $\omega_1$ , and  $\omega_2$



At  $\omega = \omega_0 = 10$  krad/s, the inductor has a value of  $j10^4 \times 10^{-2} = j100$  ohms and the capacitor has a value of  $-j10^6/10^4 = -j100$  ohms. Then,

$$V_C = -j100I$$

but

$$I = \frac{10}{10 + j100 - j100} = 1 \text{ amp}$$

So,

$$V_C = -j100 \text{ volts}$$

and

$$v_C(t) = 100 \cos(10,000t - 90^\circ) \text{ volts}$$

This gives an amplitude of **100 V** at  $\omega = \omega_0 = 10$  krad/s.

At  $\omega = \omega_1 = 9.512$  krad/s, the inductor has a value of  $j9,512 \times 0.01 = j95.12$  ohms and the capacitor has a value of  $-j10^6/9,512 = -j105.13$  ohms. Then,

$$V_C = -j105.13 I$$

but

$$I = \frac{10}{10 + j95.12 - j105.13} = \frac{10}{10 - j10.01} \text{ amps}$$

So,

$$V_C = (105.13 \angle -90^\circ)(0.7068 \angle 45.03^\circ) = 74.3 \angle -44.97^\circ \text{ volts}$$

and

$$v_C(t) = 74.3 \cos(9,512t - 44.97^\circ) \text{ volts}$$

This gives an amplitude of **74.3 V** at  $\omega = \omega_1 = 9.512$  krad/s.

At  $\omega = \omega_2 = 10.512$  krad/s, the inductor has a value of  $j10,512 \times 0.01 = j105.12$  ohms and the capacitor has a value of  $-j10^6/10,512 = -j95.13$  ohms. Then,

$$V_C = -j95.13 I$$

but

$$I = \frac{10}{10 + j105.12 - j95.13} = \frac{10}{10 + j9.99} \text{ amps}$$

So,

$$V_C = (95.13 \angle -90^\circ)(0.7075 \angle -44.97^\circ) = 67.3 \angle -134.97^\circ \text{ volts}$$

and

$$v_C(t) = 67.3 \cos(10,512t - 134.97^\circ) \text{ volts}$$

This gives an amplitude of **67.3 V** at  $\omega = \omega_2 = 10.512$  krad/s.

Note that the output voltage for this bandpass filter is the voltage across the resistor. It can be shown that

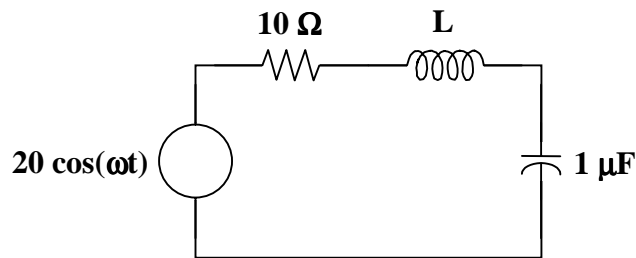
$$\begin{aligned} v_{\text{out}}(t) &= 10 \cos(10,000t) \text{ V} && \text{at } \omega = \omega_0 = 10 \text{ krad/s} \\ v_{\text{out}}(t) &= 7.068 \cos(9,512 + 45.03) \text{ V} && \text{at } \omega = \omega_1 = 9.512 \text{ krad/s} \\ v_{\text{out}}(t) &= 7.075 \cos(10,512t - 44.97^\circ) \text{ V} && \text{at } \omega = \omega_2 = 10.512 \text{ krad/s} \end{aligned}$$

The amplitude at the half-power frequencies is  $1/\sqrt{2}$  times the maximum amplitude at the center frequency. In this case,

$$\frac{1}{\sqrt{2}}(10) = 7.071$$

where the calculated amplitudes of 7.068 volts and 7.075 volts are quite close to the expected half-power value of 7.071 volts.

**Problem 14.8** Given the circuit in Figure 14.1, find the value of  $L$  so that we have a  $Q$  of 100. Also, find  $\omega_0$ ,  $\omega_1$ ,  $\omega_2$ , and  $B$ .



**Figure 14.1**

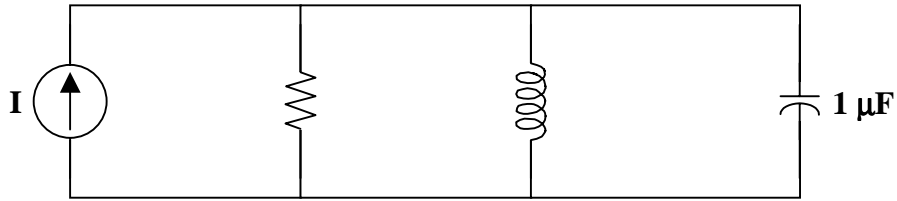
$$L = \underline{1 \text{ H}}$$

$$\omega_0 = \underline{1 \text{ krad/s}} \quad \omega_1 = \underline{995 \text{ rad/s}} \quad \omega_2 = \underline{1005 \text{ rad/s}} \quad B = \underline{10 \text{ rad/s}}$$

## PARALLEL RESONANCE

**Problem 14.9** Given the circuit in Figure 14.1 and  $I = 2\angle 0^\circ$  amps, find

- $\omega_0$ ,  $Q$ , and  $B$ ,
- $\omega_1$  and  $\omega_2$ ,
- power dissipated at  $\omega_0$ ,  $\omega_1$ , and  $\omega_2$ .



**Figure 14.1**

- (a) The resonant frequency of a parallel RLC circuit is  $\omega_0 = \frac{1}{\sqrt{LC}}$ .

So,

$$\omega_0 = \frac{1}{\sqrt{(10^{-2})(10^{-6})}} = 10^4$$

$$\omega_0 = \underline{\underline{1 \text{ krad/s}}}$$

The quality factor of a parallel RLC circuit is  $Q = \frac{R}{\omega_0 L}$ .

So,

$$Q = \frac{10^4}{10^4 \times 10^{-2}}$$

$$Q = \underline{\underline{100}}$$

The bandwidth of a parallel RLC circuit is  $B = \frac{\omega_0}{Q}$ .

So,

$$B = \frac{10^4}{100}$$

$$B = \underline{\underline{100 \text{ rad/s}}}$$

- (b) Because this is a high Q circuit, the half-power frequencies can be written as

$$\omega_{1,2} = \omega_0 \mp \frac{B}{2}$$

So,

$$\omega_{1,2} = 10^4 \mp \frac{100}{2}$$

$$\omega_{1,2} = 10^4 \mp 50$$

or

$$\omega_1 = \underline{\underline{9.95 \text{ krad/s}}}$$

$$\omega_2 = \underline{\underline{10.05 \text{ krad/s}}}$$

- (c) Find the power dissipated at  $\omega_0 = 1 \text{ krad/s}$ .

Since all of the current flows through the resistor at resonance,  $P = \frac{1}{2} |I|^2 R$ .

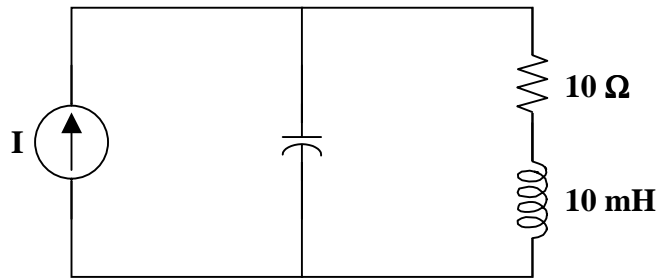
So,

$$P = \frac{1}{2} (2)^2 (10^4)$$

$$P = \underline{\mathbf{20 \text{ kwatts}}}$$

Since  $\omega_1$  and  $\omega_2$  correspond to the half-power points, the power dissipated at  $\omega_1$  and  $\omega_2$  is **10 kwatts**.

**Problem 14.10** Given the circuit in Figure 14.1, find the resonant frequency.



**Figure 14.1**

Begin by finding the parallel equivalent of the series resistor and inductor elements. The parallel equivalent is given by

$$\frac{1}{R + j\omega L} = \frac{1}{R_{eq}} + \frac{1}{jX_{eq}}$$

So,

$$\frac{1}{10 + j\omega/100} = \frac{1}{R_{eq}} + \frac{1}{jX_{eq}}$$

$$\frac{10 - j\omega/100}{100 + (\omega/100)^2} = \frac{1}{R_{eq}} + \frac{1}{jX_{eq}}$$

Thus,

$$\frac{-j\omega/100}{100 + (\omega/100)^2} = \frac{1}{jX_{eq}}$$

$$X_{eq} = \frac{100 + (\omega/100)^2}{\omega/100}$$

$$X_{eq} = \frac{10^4}{\omega} + \frac{\omega}{100}$$

At resonance,

$$\omega = \omega_0 \quad \text{and} \quad X_{eq} = X_C$$

where

$$X_C = \frac{1}{\omega C} = \frac{1}{\omega 10^{-6}} = \frac{10^6}{\omega}$$

Thus,

$$\frac{10^4}{\omega_0} + \frac{\omega_0}{100} = \frac{10^6}{\omega_0}$$

or

$$10^4 + \frac{\omega_0^2}{100} = 10^6$$

$$\omega_0^2 = 10^8 - 10^6 = 9.9 \times 10^7$$

$$\omega_0 = \underline{\underline{9.95 \text{ krad/s}}}$$

or

$$\omega_0 \cong \underline{\underline{10 \text{ krad/s}}}$$



## PASSIVE FILTERS

**Problem 14.11** What type of filter is represented by the circuit in Figure 14.1? What is the cutoff frequency, or what are the corner frequencies?

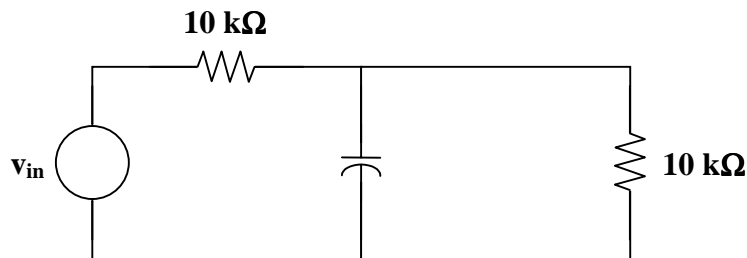
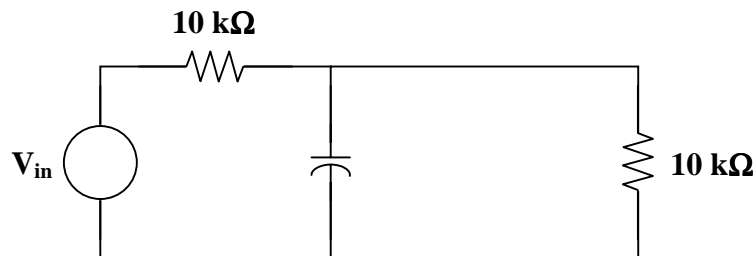


Figure 14.1

In the frequency domain, the circuit is



Find the transfer function

$$H(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}}$$

Using nodal analysis,

$$\frac{V_{\text{out}} - V_{\text{in}}}{10^4} + \frac{V_{\text{out}}}{-j10^5/\omega} + \frac{V_{\text{out}}}{10^4} = 0$$

Simplifying,

$$V_{\text{out}} - V_{\text{in}} + j\frac{\omega}{10} V_{\text{out}} + V_{\text{out}} = 0$$
$$\left(2 + j\frac{\omega}{10}\right) V_{\text{out}} = V_{\text{in}}$$

Hence,

$$H(\omega) = \frac{1}{2 + j\omega/10}$$

This transfer function looks like a typical transfer function for a lowpass filter

$$\frac{1}{1 + j\omega RC}$$

Since the voltage starts at  $V_{\text{out}} = \frac{1}{2} V_{\text{in}}$  and as  $\omega \rightarrow \infty$   $V_{\text{out}} = 0$ , we can look at a value for

$$|V_{\text{out}}| = \left(\frac{0.7071}{2}\right) |V_{\text{in}}| = 0.3535 |V_{\text{in}}|$$

To find the cutoff frequency, find the value of  $\omega$  when  $|H(\omega)| = 0.3535$ .

$$|H(\omega)| = \frac{1}{\sqrt{4 + \frac{\omega^2}{100}}} = 0.3535$$

$$8 = 4 + \frac{\omega^2}{100}$$

$$\omega^2 = 400$$

$$\omega = 20 \text{ rad/s}$$

This **lowpass filter** has a cutoff frequency of  $\omega_c = \underline{20 \text{ rad/s}}$  or  $f_c = \underline{10/\pi \text{ Hz}}$ .



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**Problem 14.12** [14.43] Determine the range of frequencies that will be passed by a series RLC bandpass filter with  $R = 10\ \Omega$ ,  $L = 25\ \text{mH}$ , and  $C = 0.4\ \mu\text{F}$ . Find the quality factor.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25 \times 10^{-3})(0.4 \times 10^{-6})}} = 10\ \text{krad/s}$$

$$B = \frac{R}{L} = \frac{10}{0.025} = 400\ \text{rad/s}$$

Thus,

$$Q = \frac{\omega_o}{B} = \frac{10}{0.4}$$

$$Q = \underline{\underline{25}}$$

This is a high Q circuit so we can use

$$\omega_1 = \omega_o - B/2 = 10 - 0.2 = 9.8\ \text{krad/s} \quad \text{or} \quad f_1 = \frac{9.8}{2\pi} = 1.5597\ \text{kHz}$$

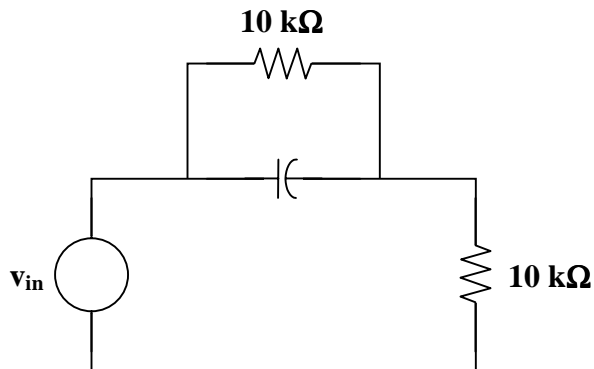
$$\omega_2 = \omega_o + B/2 = 10 + 0.2 = 10.2\ \text{krad/s} \quad \text{or} \quad f_2 = \frac{10.2}{2\pi} = 1.6234\ \text{kHz}$$

Therefore,

$$\underline{\underline{1.5597\ \text{kHz} < f < 1.6234\ \text{kHz}}}$$


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**Problem 14.13** What type of filter is represented by the circuit in Figure 14.1? What is the cutoff frequency, or what are the corner frequencies?



**Figure 14.1**

This highpass filter has a cutoff frequency of  $\omega_c = \underline{\underline{26.55\ \text{rad/s}}}$  or  $f_c = \underline{\underline{4.226\ \text{Hz}}}$ .

## ACTIVE FILTERS

**Problem 14.14** What type of filter is represented by the circuit in Figure 14.1? What is the cutoff frequency, or what are the corner frequencies?

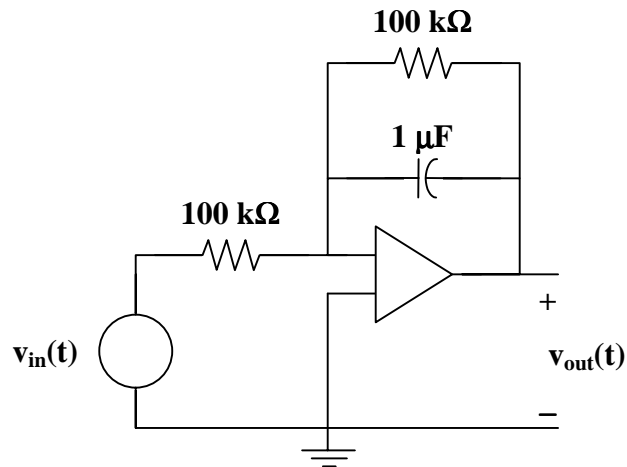
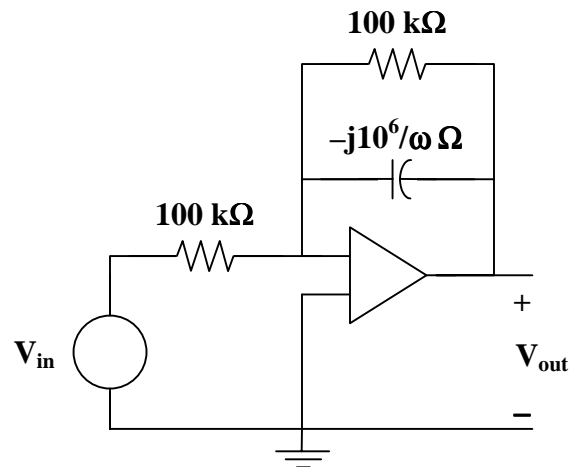


Figure 14.1

In the frequency domain, the circuit is



Find the transfer function

$$H(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}}$$

Using nodal analysis,

$$\frac{V_a - V_{in}}{10^5} + \frac{V_a - V_{out}}{10^5} + \frac{V_a - V_{out}}{-j10^6/\omega} = 0$$

where  $V_a = V_b = 0$ .

Simplifying,

$$-V_{in} - V_{out} - j\frac{\omega}{10}V_{out} = 0$$

$$\left(1 + j\frac{\omega}{10}\right)V_{out} = -V_{in}$$

Hence,

$$H(\omega) = \frac{1}{1 + j\omega/10}$$

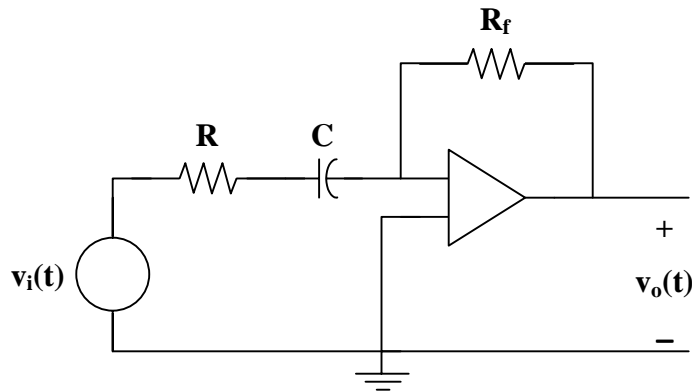
This transfer function looks like a typical transfer function for a lowpass filter

$$\frac{1}{1 + j\omega RC}$$

This **lowpass filter** has a cutoff frequency of  $\omega_c = \underline{10 \text{ rad/s}}$  or  $f_c = \underline{1.5915 \text{ Hz}}$ .

**Problem 14.15** [14.55] Design the filter in Figure 14.1 to meet the following requirements :

- It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.
- It must provide a steady-state output of  $v_o(t) = 10\sin(2\pi \times 10^8 t + 180^\circ)$  volts for an input  $v_i(t) = 4\sin(2\pi \times 10^8 t)$  volts.



**Figure 14.1**

This is a highpass filter with

$$f_c = 2 \text{ kHz}, \quad \omega_c = 2\pi f_c = 1/(RC)$$

or

$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

Clearly, the capacitor becomes a short circuit at high frequencies. Hence, the high frequency gain is

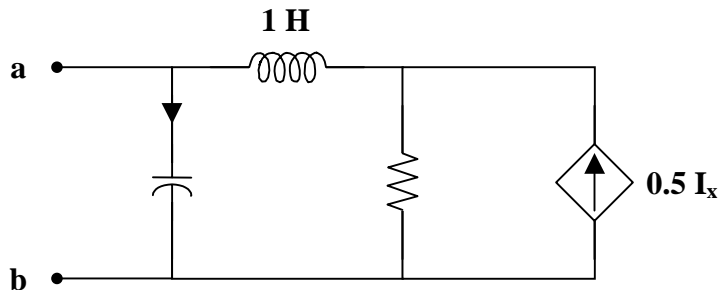
$$\frac{-R_f}{R} = \frac{-10}{4} \quad \text{or} \quad R_f = 2.5R$$

If we let  $R = \underline{10 \text{ k}\Omega}$ , then  $R_f = \underline{25 \text{ k}\Omega}$ , and  $C = \frac{1}{4000\pi \times 10^4} = \underline{7.958 \text{ nF}}$ .

## SCALING

**Problem 14.16** [14.63] For the circuit in Figure 14.1,

- draw the new circuit after it has been scaled by  $K_m = 200$  and  $K_f = 10^4$ .
- obtain the Thevenin equivalent impedance at terminals a-b of the scaled circuit at  $\omega = 10^4$  rad/s.



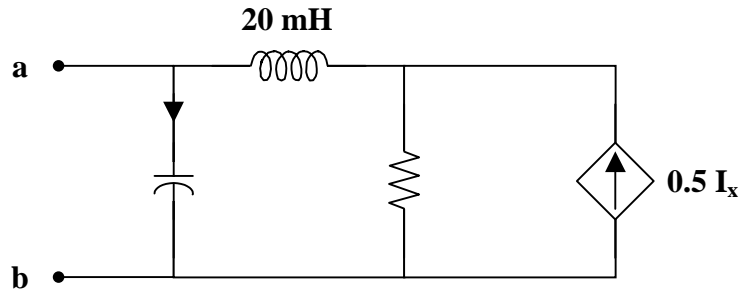
**Figure 14.1**

$$(a) \quad R' = K_m R = (200)(2) = 400 \, \Omega$$

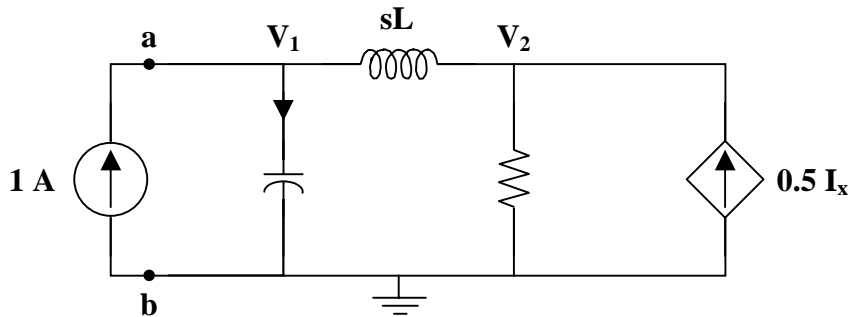
$$L' = \frac{K_m L}{K_f} = \frac{(200)(1)}{10^4} = 20 \text{ mH}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.5}{(200)(10^4)} = 0.25 \, \mu\text{F}$$

We now have a new circuit.



(b) Insert a 1 amp source at the terminals a-b.



At node 1 :

$$1 = \frac{V_1}{1/(sC)} + \frac{V_1 - V_2}{sL}$$

At node 2 :

$$\frac{V_1 - V_2}{sL} + 0.5 I_x = \frac{V_2}{R}$$

But,  $I_x = sC V_1$ .

So, the nodal equations become

$$1 = sC V_1 + \frac{V_1 - V_2}{sL}$$

$$\frac{V_1 - V_2}{sL} + 0.5sC V_1 = \frac{V_2}{R}$$

Solving for  $V_1$ ,

$$V_1 = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

$$Z_{Th} = \frac{V_1}{1} = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

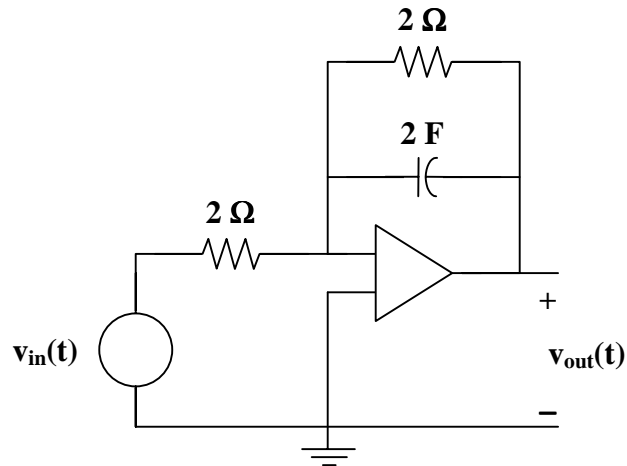
$$\text{At } \omega = 10^4, \quad Z_{Th} = \frac{(j10^4)(20 \times 10^{-3}) + 400}{(j10^4)^2(20 \times 10^{-3})(0.25 \times 10^{-6}) + 0.5(j10^4)(0.25 \times 10^{-6})(400) + 1}$$

$$Z_{Th} = \frac{400 + j200}{0.5 + j0.5} = 600 - j200$$

$$Z_{Th} = \underline{\underline{632.5 \angle -18.43^\circ \Omega}}$$

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**Problem 14.17** Given the circuit in Figure 14.1, find the values necessary to scale this circuit, increasing the corner frequency to 100 rad/s. Use a 1  $\mu\text{F}$  capacitor.



**Figure 14.1**

To scale the circuit in Figure 14.1 from  $\omega = 1/4$  rad/s to  $\omega' = 100$  rad/s using a 1  $\mu\text{F}$  capacitor, **the feedback resistor and the input resistor must be 10 k $\Omega$ .**